How quantum algorithms use interference

A marked phase becomes a larger probability only through a carefully designed transformation.

14 min read Dr. Charith Jayasekara

By the end, you'll be able to

- Trace one exact two-qubit Grover iteration.
- Explain why too many amplification iterations reduce success.

A complete four-item example

Prepare the uniform state with amplitudes $(1/2, 1/2, 1/2, 1/2)$. Suppose item 11 is marked. Its phase oracle produces $(1/2, 1/2, 1/2, -1/2)$.

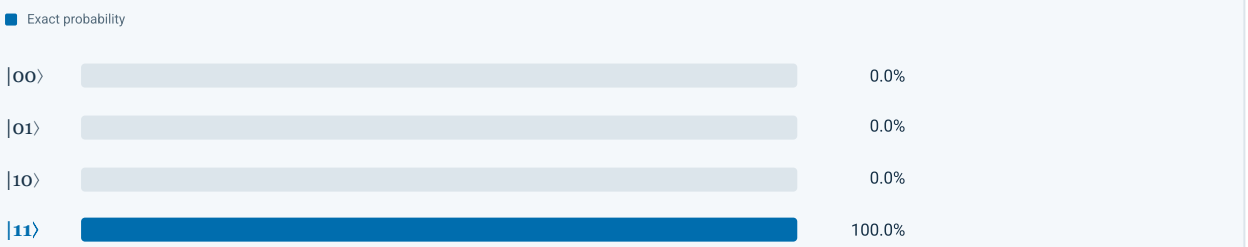
The mean amplitude is now $1/4$. Diffusion reflects each amplitude a about the mean: $a \mapsto 2(1/4) - a$. The first three become 0 and the marked amplitude becomes 1. One ideal iteration therefore finds 11 with certainty in this special four-item case.

L06 · INTERACTIVE LAB

Runs on your device

Grover search explorer

An oracle flips the marked amplitude. Diffusion reflects amplitudes about their mean. Repetition first increases and then decreases the chance of success.



Marked item probability **100.0%**

MODEL · Ideal pure-state vector, complex double precision ORDER · q0 is the leftmost bit

Prediction: There are four possible items and one marked item. How many Grover iterations will you need? Challenge: Choose 3 qubits. Increase the iterations past the best point and explain why the success probability falls. Open the live experiment studio.

The general pattern

For one marked item among N, let $\sin\theta = 1/\sqrt{N}$. After k Grover iterations, success probability is

$$\sin^2((2k+1)\theta).$$

This oscillates. Increasing k forever does not improve the result. A useful iteration count is near $\pi\sqrt{N}/4$, with rounding and marked-set assumptions handled carefully.

What the lab verifies

The model uses one marked basis state, exact phase marking and exact diffusion. It counts one oracle query per iteration but excludes the cost of constructing that oracle, preparing application data and executing fault-tolerant gates.

A classical simulator calculates all these amplitudes explicitly. Its wall-clock time is not a measurement of a quantum device's runtime.

Challenge: for eight items, calculate the probabilities at k = 0, 1, 2 and 3. Compare them with the bars, then explain the drop after the best point using the angle formula.

Reference: IBM Quantum — Fundamentals of quantum algorithms.