Quantum oracles and phase kickback

A reversible function evaluation can become a phase that interference can read.

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By the end, you'll be able to

- Describe a reversible Boolean oracle.
- Derive phase kickback using a minus-state target.

Make a function reversible

A Boolean function $f(x)$ may discard information. Its reversible oracle keeps the input and XORs the output into a target:

$$U_f|x, y\rangle = |x, y \oplus f(x)\rangle.$$

This map is its own inverse. The oracle is a specified operation, not a magical free answer source; its implementation can dominate an algorithm's cost.

Use an eigenstate of X

The state $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$ satisfies $X|-\rangle = -|-\rangle$.

So a function-controlled X on this target gives

$$U_f\big(|x\rangle|-\rangle\big) = (-1)^{f(x)}|x\rangle|-\rangle.$$

The target factors out; the input receives a function-dependent phase. This is **phase kickback**.

A two-input example

Let $f(x_0x_1) = x_1$. The values on 00, 01, 10 and 11 are 0, 1, 0 and 1. A uniform input register receives signs $+, -, +, -$. Applying H to both input qubits converts that phase pattern into a nonzero bitstring.

The Deutsch–Jozsa lab calculates this phase-kickback input register directly. It explicitly factors out the minus-state ancilla; it does not pretend to show an extra simulated wire.

Check yourself: if f is constant one, every component acquires the same minus sign. Why is this global phase unobservable, while a balanced pattern is not?

Reference: IBM Quantum — Fundamentals of quantum algorithms.