Controlled gates and CNOT

Conditional quantum operations work coherently, without first measuring the control.

10 min read Dr. Charith Jayasekara

By the end, you'll be able to

- Trace CNOT on basis states and superpositions.
- State the control, target and bit-order convention.

A reversible truth table

Controlled-X, often called CNOT or CX, maps

$$|c, t\rangle \longmapsto |c, t \oplus c\rangle.$$

Input	Output
00	00
01	01
10	11
11	10

In this book's playground, q0 is the control and q1 the target. CX preserves q0 and flips q1 only on the branch where q0 is 1.

No intermediate measurement

For a superposed control, linearity applies the operation to both components coherently:

$$\frac{|00\rangle + |10\rangle}{\sqrt{2}} \longmapsto \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

This creates entanglement for that input. CX does not entangle every input: $|00\rangle$ remains a product state, and a target in $|+\rangle$ is unchanged by X.

Reverse it

Applying the same CX twice restores the input because XOR with the same control twice cancels. The gate is unitary and self-inverse.

Practice: prepare H on q1 only, then CX. Predict the four amplitudes. Compare it with H on q0 followed by CX. The difference is a useful way to catch swapped control and target conventions.

Reference: IBM Quantum — Multiple systems.