Phase and rotation gates

Continuous changes of angle, and the phase conventions behind familiar gate names.

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By the end, you'll be able to

- Relate S and T to the phase gate P.
- Calculate a Y rotation on the zero state.

A relative phase gate

$$P(\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}.$$

It leaves the magnitude of each amplitude unchanged. Therefore it does not change Z-basis probabilities immediately. S is $P(\pi/2)$, T is $P(\pi/4)$ and Z is $P(\pi)$.

The interference experiment places P between two H gates so you can observe its effect.

A rotation is related, but check its convention

$$R_z(\phi) = \begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix}.$$

For a standalone one-qubit operation, $P(\phi) = e^{i\phi/2} R_z(\phi)$, so they differ only by global phase. Controlled versions require more care: a phase on one controlled branch can become a relative phase in the larger system.

The Y rotation is

$$R_y(\theta) = \begin{pmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{pmatrix}.$$

Applying it to zero gives amplitudes $\cos(\theta/2)$ and $\sin(\theta/2)$. At $\theta = 2\pi/3$, the probabilities are $1/4$ and $3/4$.

Practise

Set the qubit explorer's polar angle to 120° and phase to 0°. Compare it with the Y-rotation example. Then apply S and T and explain why the displayed Z probabilities stay fixed while the arrow moves.

Reference: IBM Quantum — Single systems.