Pauli gates and the Hadamard gate

Bit flips, phase flips and a change of basis.

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By the end, you'll be able to

- Calculate the effect of X, Y, Z and H on basis states.
- Verify HZH = X using a worked input.

The Pauli matrices

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

X exchanges $|0\rangle$ and $|1\rangle$. Y sends $|0\rangle$ to $i|1\rangle$ and $|1\rangle$ to $-i|0\rangle$. Z leaves $|0\rangle$ alone and negates $|1\rangle$.

Each Pauli squares to I. On a basis state, Z's minus may be only a global phase; on a superposition it can change a relative phase.

Hadamard changes the basis

$$H|0\rangle = |+\rangle, \qquad H|1\rangle = |-\rangle.$$

Because $H^2 = I$, it also maps $|+\rangle$ back to 0 and $|-\rangle$ back to 1. This is why H before Z readout implements an X-basis measurement.

A worked identity

Trace $HZH|0\rangle$: the first H gives plus, Z gives minus, and the second H gives 1. On $|1\rangle$, the sequence gives 0. Thus $HZH = X$ on a basis, hence on all states by linearity.

Practise

Use the qubit explorer to compare X and Y on $|0\rangle$. Their outcome probabilities agree, but their statevectors differ by a global phase. Then test them on a superposition: agreement on one input is not agreement as operators.

Reference: IBM Quantum — Single systems.