Quantum reversibility and the no-cloning principle

Ideal gates are reversible. Copying an arbitrary unknown state is a different request.

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By the end, you'll be able to

- Reverse a sequence of unitary gates.
- Give the inner-product argument against universal perfect cloning.

Undoing a gate

An ideal unitary U has inverse $U^\dagger$. To reverse a sequence, reverse its order and invert each gate. If a circuit applies H, then S, its state is $SH|\psi\rangle$. Undo it by applying $S^\dagger$ first and H second.

X, Z, H and CX are their own inverses. S is not: $S^\dagger = \mathrm{diag}(1, -i)$.

Copying known basis information

CX maps $|0\rangle|0\rangle$ to $|00\rangle$ and $|1\rangle|0\rangle$ to $|11\rangle$. This copies a known computational-basis label into a fresh zero target.

For a superposition, however,

$$CX\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |0\rangle\right) = \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

That is an entangled Bell state, not $|+\rangle \otimes |+\rangle$.

Why no universal perfect cloner exists

Suppose a unitary copied both unknown states $|a\rangle$ and $|b\rangle$ into a blank target. Preservation of inner products would require

$$\langle a|b\rangle = (\langle a|b\rangle)^2.$$

This only holds when the overlap is 0 or 1. It fails for general distinct, nonorthogonal states. A single universal operation therefore cannot perfectly clone arbitrary unknown pure states. This does not forbid preparing many copies of a state whose description you already know.

Practise

Prepare a Bell state in the gate playground. Undo CX and then H. Verify that all probability returns to 00. Explain why measurement halfway through would change this reversal experiment.

Reference: IBM Quantum — Basics of quantum information.