Entanglement and Bell states

A joint quantum state can contain relationships no product of individual states can describe.

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By the end, you'll be able to

- Construct a Bell state using H and CX.
- Distinguish entanglement from independent states and classical correlation.
- Explain why entanglement does not send a message faster than light.

Build a Bell pair

Start from $|00\rangle$. Apply H to q0, then controlled-X with q0 as control:

$$|00\rangle \xrightarrow{H_0} \frac{|00\rangle + |10\rangle}{\sqrt{2}} \xrightarrow{CX_{0,1}} |\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

The Z-basis outcomes are 00 and 11, each with probability $1/2$. Each individual outcome is random, but the pair agrees.

Why it is not a product

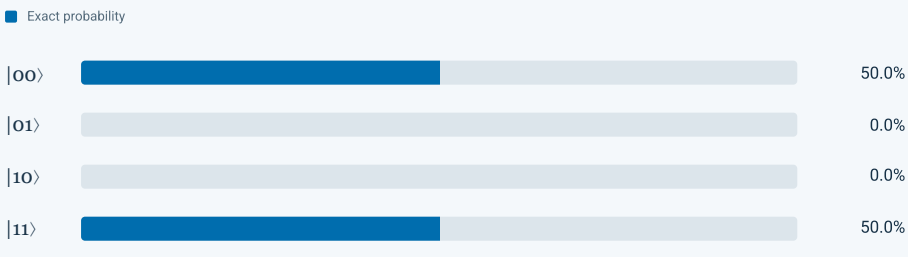
Suppose $(a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle)$ equalled this Bell state. We would need ac and bd nonzero, but $ad = bc = 0$. Nonzero ac and bd require all four factors nonzero, contradicting those zeros. The state is not separable.

L04 · INTERACTIVE LAB

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Entanglement explorer

Prepare (|00⟩ + |11⟩)/√2 using H and CX. Compare this with independent superpositions in two measurement bases.



MODEL · Ideal pure-state vector, complex double precision ORDER · q0 is the leftmost bit

Prediction: The Bell state agrees in Z. Will it still agree if both qubits are measured in X? Challenge: Compare the Bell state with two independent |+⟩ states. Why are correlations in one basis alone insufficient to establish entanglement? Open the live experiment studio.

One basis is not enough evidence

A classical 50/50 mixture of 00 and 11 has the same Z-basis counts as this Bell state. Their X-basis behaviour differs: the ideal Bell state still has matching outcomes, while that classical mixture does not preserve those X correlations.

The lab compares Bell preparation with independent $|+\rangle$ states. This distinguishes those two known ideal preparations; it is not a general experimental certification of an arbitrary unknown device. Such a claim needs carefully chosen observables and assumptions.

Correlation is not communication

Measuring one half gives a random local outcome. The other reader cannot use their own outcomes to learn which measurement was chosen remotely. Comparing correlations requires ordinary classical communication.

Challenge: predict the independent-state distributions in Z and X before switching the selector. Explain both using the product-state structure.

Reference: Microsoft Learn — Entanglement and correlations.