Multiple qubits and tensor products

Combining two systems creates four amplitudes, with an ordering convention that matters.

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By the end, you'll be able to

- Compute a two-qubit product state numerically.
- Use the site's most-significant-bit ordering convention.

Combine the components

For $|a\rangle = (a_0, a_1)^T$ and $|b\rangle = (b_0, b_1)^T$,

$$|a\rangle \otimes |b\rangle = \begin{pmatrix} a_0 b_0 \\ a_0 b_1 \\ a_1 b_0 \\ a_1 b_1 \end{pmatrix}.$$

Our basis order is $|00\rangle, |01\rangle, |10\rangle, |11\rangle$. The leftmost qubit, q0, is the most significant bit. Other frameworks may display bitstrings differently; check before comparing results.

A complete example

Take $|a\rangle = |+\rangle$ and $|b\rangle = |1\rangle$. Their joint vector is

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \frac{|01\rangle + |11\rangle}{\sqrt{2}}.$$

Measuring gives 01 or 11 equally often. The second qubit is always 1; the first is equally likely to be 0 or 1.

More states, more memory

An n-qubit pure state has 2^n complex amplitudes. A general classical statevector simulation therefore needs exponential memory. This describes the cost of storing an arbitrary statevector, not a claim that a quantum device lets you read all those amplitudes in one shot.

Some structured states and circuits admit much more efficient classical simulation. Exponential state-space dimension alone is not evidence of useful quantum advantage.

Practise

In the gate playground, apply H to q0 and X to q1. Verify the expected 01/11 distribution. Then change X to q0 instead: explain the result by tracing the ordering rather than guessing from the bitstring labels.

Reference: IBM Quantum — Multiple systems.