Quantum phase and interference

The invisible difference that another gate can make visible.

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By the end, you'll be able to

- Distinguish global phase from relative phase.
- Derive the output of an H–phase–H circuit.

Global phase changes no prediction

Multiplying an entire state by $e^{i\gamma}$ does not change measurement probabilities. For any measurement vector $|v\rangle$,

$$|\langle v|e^{i\gamma}\psi\rangle|^2 = |e^{i\gamma}|^2|\langle v|\psi\rangle|^2 = |\langle v|\psi\rangle|^2.$$

Relative phase is different. Multiplying just one component changes its relationship with the others.

Work the circuit through

Prepare $|0\rangle$, apply H, then $P(\phi) = \mathrm{diag}(1, e^{i\phi})$, then H again:

$$|0\rangle \longmapsto \frac{|0\rangle + |1\rangle}{\sqrt{2}} \longmapsto \frac{|0\rangle + e^{i\phi}|1\rangle}{\sqrt{2}} \longmapsto \frac{1+e^{i\phi}}{2}|0\rangle + \frac{1-e^{i\phi}}{2}|1\rangle.$$

The final probabilities are

$$P(0) = \cos^2(\phi/2), \qquad P(1) = \sin^2(\phi/2).$$

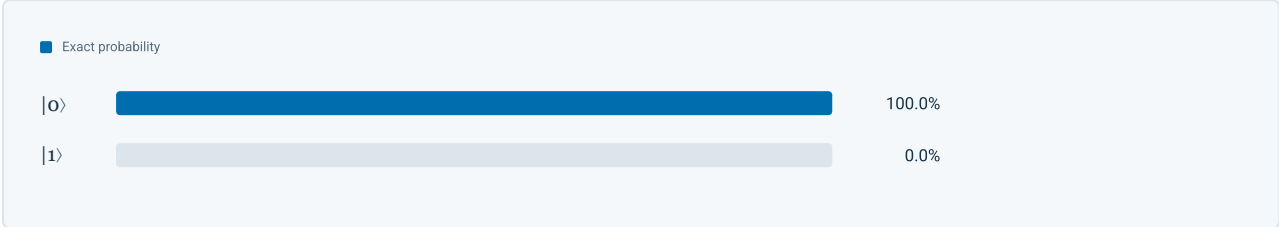
Before the second H, the Z probabilities are always 50/50. The final H recombines amplitudes and exposes the phase difference.

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Superposition & interference

The two Hadamard gates recombine amplitudes. A relative phase between them changes the result even though the intermediate Z-basis probabilities stay at 50/50.



MODEL · Ideal pure-state vector, complex double precision ORDER · q0 is the leftmost bit

Prediction: H then H returns 0. What do you expect if a Z gate is placed between them? Challenge: Set the phase to 180°. Explain why the final probability of 1 becomes 100%. Open the live experiment studio.

Three checks

- At 0°, the amplitudes for 1 cancel; the result is 0.
- At 180°, the amplitudes for 0 cancel; the result is 1.
- At 90°, the output probabilities are equal.

Challenge: explain why applying a global minus sign at the middle of the circuit cannot replace the Z gate. Z changes the relative sign; a global minus multiplies both amplitudes.

Reference: IBM Quantum — Single systems.