Probability, amplitudes and normalisation

Probabilities add to one. Amplitudes can do something more interesting.

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By the end, you'll be able to

- Convert complex amplitudes into measurement probabilities.
- Validate and normalise a nonzero state vector.

The rule that connects state to outcome

For $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, the Born rule gives $P(0) = |\alpha|^2$ and $P(1) = |\beta|^2$. The state must be normalised:

$$|\alpha|^2 + |\beta|^2 = 1.$$

An amplitude can be negative or complex. A probability is real and lies between zero and one.

A complete numerical example

Take amplitudes $\alpha = 1/2$ and $\beta = i\sqrt{3}/2$. Their squared magnitudes are $1/4$ and $3/4$, so the vector is normalised. Across 1,000 independent preparations, the **expected** counts are 250 and 750. A sampled result will usually differ.

If instead someone gives you the vector $(1, i)^T$, its squared norm is 2. Divide both amplitudes by $\sqrt{2}$ to obtain a valid unit vector. You cannot normalise the zero vector because its norm is zero.

Add amplitudes before squaring

When indistinguishable paths contribute to the same outcome, their amplitudes add. Squaring afterwards introduces interference terms:

$$|a + b|^2 = |a|^2 + |b|^2 + 2\,\mathrm{Re}(a^* b).$$

For $a = 1/2$ and $b = -1/2$, the total amplitude is zero. Adding the two separate squared magnitudes would incorrectly give $1/2$.

PAUSE & THINK

A qubit has amplitudes 1/2 and i√3/2. What is P(1)?

- [ ] √3/2
- [ ] 3/4
- [ ] 1/2

Practise

In the qubit explorer, find a state with $P(1) = 3/4$. Change only its phase. Then switch to the interference lab and explain why relative phase can affect a later measurement even when the current Z probabilities do not change.

Reference: IBM Quantum — Quantum information.