Dirac notation — kets and bras

A compact language for state vectors, inner products and projections.

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By the end, you'll be able to

- Translate between column vectors and ket notation.
- Compute a bra-ket inner product with complex conjugation.

A ket is a column

$|\psi\rangle$ means a state vector. For one qubit,

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}.$$

Its corresponding **bra** is the conjugate-transposed row:

$$\langle\psi| = (\alpha^*, \beta^*).$$

Complex conjugation matters. If $\beta = i/\sqrt{2}$, its conjugate is $-i/\sqrt{2}$.

Inner products

The expression $\langle\phi|\psi\rangle$ is a complex number. It describes overlap between vectors. The basis states are orthonormal: $\langle 0|0\rangle = 1$ and $\langle 0|1\rangle = 0$.

For $|\psi\rangle = (|0\rangle + i|1\rangle)/\sqrt{2}$,

$$\langle\psi|\psi\rangle = \frac{1}{2} + \frac{(-i)i}{2} = 1.$$

The probability of a Z-basis outcome 1 is $|\langle 1|\psi\rangle|^2 = 1/2$.

Outer products are operators

$|0\rangle\langle 0|$ is a matrix, not a scalar:

$$|0\rangle\langle 0| = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}.$$

It projects onto the $|0\rangle$ direction. The distinction is visual: a bra followed by a ket gives an inner product; a ket followed by a bra gives an operator.

Check yourself: calculate $\langle +|-\rangle$. The two terms cancel, giving zero, even though their Z-basis probabilities are identical.

Reference: IBM Quantum — Single systems.