# Vectors, matrices and linear algebra

Quantum states are vectors. Gates are matrices. A small calculation connects the two.

12 min read Dr. Charith Jayasekara

**By the end, you'll be able to**

- Multiply a two-by-two matrix by a state vector.
- Explain norm preservation for an ideal quantum gate.

## A state as a column

We represent the basis states with two simple columns:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

A general pure qubit has column $(\alpha, \beta)^T$. Its squared norm is $|\alpha|^2 + |\beta|^2 = 1$.

## A gate as a matrix

The X gate exchanges the two amplitudes:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad X \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}.$$

To multiply a matrix by a column, take each row's dot product with the column. For the first row above, $0\alpha + 1\beta = \beta$.

The Hadamard gate is

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Applying it to $|0\rangle$ gives $(1, 1)^T/\sqrt{2}$. Applying it again gives $(1, 0)^T$. The cancellation in the second component is interference in matrix form.

## Why unitary matrices?

An ideal closed-system gate $U$ satisfies $U^\dagger U = I$, where the dagger means conjugate transpose. This property preserves inner products and norms, so valid states remain normalised. It also means $U^{-1} = U^\dagger$.

Measurement and noisy evolution need more general descriptions; do not assume every physical operation is a unitary gate on the subsystem you observe.

## Practise

Calculate $H|1\rangle$. You should obtain $(1, -1)^T/\sqrt{2}$. Then verify $X^2 = I$ and $H^2 = I$ directly. Use the gate playground to compare the numerical outputs with your calculation.

Reference: IBM Quantum — Single systems.