Essential mathematics — complex numbers

Two real numbers, one amplitude, and a useful way to describe phase.

10 min read Dr. Charith Jayasekara

By the end, you'll be able to

- Add and multiply simple complex numbers.
- Calculate squared magnitude and distinguish it from an amplitude.

Meet the imaginary unit

The number i satisfies $i^2 = -1$. A complex number $z = a + bi$ has a real part a and an imaginary part b. You can picture it as the point (a, b) on a plane.

Addition works component by component:

$$(1 + 2i) + (3 - i) = 4 + i.$$

Multiplication uses ordinary expansion and $i^2 = -1$:

$$(1 + i)(1 - i) = 1 - i + i - i^2 = 2.$$

Squared magnitude becomes probability

The conjugate of $z = a + bi$ is $z^* = a - bi$. The squared magnitude is

$$|z|^2 = z^* z = a^2 + b^2.$$

If an amplitude is $\alpha = (1 + i)/2$, then $|\alpha|^2 = (1/2)^2 + (1/2)^2 = 1/2$. It is the **squared magnitude**, not the real part and not the amplitude itself, that becomes a basis-measurement probability.

Phase is direction

A unit-magnitude complex number can be written

$$e^{i\phi} = \cos\phi + i\sin\phi.$$

Multiplication by this number rotates an amplitude in the complex plane. For example, multiplying 1 by i rotates by $\pi/2$ radians, or 90 degrees. Its magnitude stays one.

In a quantum state, relative phase between amplitudes affects interference. Multiplying the **whole state** by the same unit phase does not change physical predictions.

Work it through

For $|\psi\rangle = (\sqrt{3}/2)|0\rangle + (i/2)|1\rangle$, the probabilities are $3/4$ and $1/4$. They sum to one. Replacing $i/2$ with $-i/2$ keeps those Z probabilities but changes the relative phase.

Try it: calculate $|3+4i|^2$, then divide $3+4i$ by 5. The results are 25 and a unit-magnitude complex number, respectively.

Reference: IBM Quantum — Single systems.